

5 Pre-dominantly  $J^{PC} = 1^{--}$  states are produced, because the photon has a  $J^{PC} = 1^{--}$  state.

(b)  $-1 = P = P(q)P(\bar{q})(-1)^L = (-1)^{L+1} \Rightarrow L \in \{0, 2, 4, \dots\}$  ✓  
 $-1 = C = (-1)^{L+S} \Rightarrow L+S = \text{odd}$   
 $\vec{J} = \vec{L} + \vec{S}$  Since  $J=1$  and  $S = 2 \cdot \frac{1}{2} + 1 = 2$ ,  $L$  must be 2. ???

- $L=0, S=2 \Rightarrow J \in \{2\}$
- $L=2, S=2 \Rightarrow J \in \{0, 1, 2, 3, 4\}$  ← only one containing  $J=1$
- $L=4, S=2 \Rightarrow J \in \{2, \dots, 6\}$

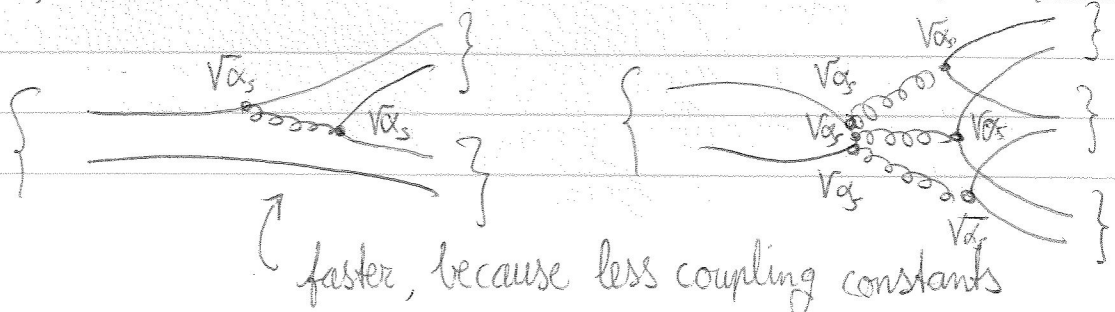
(c)  $\Upsilon$  has <sup>no</sup> colour charge (it is a ~~meson~~ <sup>particle-antiparticle configuration</sup>), whereas the gluon carries colour charge.

(d) Estimation of the lifetime:  $\tau \approx \frac{1}{\Gamma}$  in natural units

$\Upsilon(1S, 2S, 3S)$ :  $\tau = \frac{1}{54 \text{ keV}} \approx 1.9 \times 10^{-5} \text{ eV}^{-1}$  till  $\tau = \frac{1}{20 \text{ keV}} = 5.0 \times 10^{-5} \text{ eV}^{-1}$

$\Upsilon(4S)$ :  $\tau = \frac{1}{20 \text{ MeV}} = 5.0 \times 10^{-7} \text{ eV}^{-1}$

The lightest open-bottom meson ( $B$ ) is 5.3 GeV, that means that ~~heavier~~ for ~~more~~ heavier mesons this open-bottom threshold is passed and the much faster decay via one gluon is possible. Therefore the lifetimes of these states are shorter, which results in an increase in the width (OZI rule)



②  $\mathcal{L} = \frac{1}{2} \partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi)$ , where  $V(\phi^* \phi) = \frac{\mu^2}{2} \phi^* \phi + \frac{\lambda}{4} (\phi^* \phi)^2$

(a)  $\mu^2 < 0, \lambda > 0$  write the field  $\phi$  in polar coordinates:  $\phi = \rho e^{i\alpha}$   
 then  $\phi^* \phi = \rho e^{-i\alpha} \cdot \rho e^{i\alpha} = \rho^2$ . Now, the potential can be written as

$$V(\rho^2) = \frac{\mu^2}{2} \rho^2 + \frac{\lambda}{4} (\rho^2)^2 = \frac{\rho^2}{2} (\mu^2 + \frac{\lambda}{2} \rho^2)$$

Its derivative with respect to  $\rho$  is

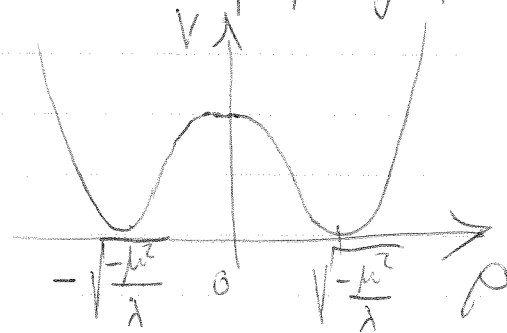
$$\frac{\partial V(\rho^2)}{\partial \rho} = \rho (\mu^2 + \lambda \rho^2)$$

Σ 15  
 ②

5 It is zero when  $\rho = 0$  or  $\rho = \pm \sqrt{\frac{-\mu^2}{\lambda}}$  (which is real when  $\mu^2 < 0$  and  $\lambda > 0$ )

So, it has multiple distinct real extrema, which is a property of SSB.

Mexican hat potential



(b)  $\partial^\mu \phi = \partial^\mu (\rho e^{i\alpha}) = e^{i\alpha} \partial^\mu \rho + i \rho e^{i\alpha} \partial^\mu \alpha$   
 $\partial_\mu \phi^* = \dots = e^{-i\alpha} \partial_\mu \rho - i \rho e^{-i\alpha} \partial_\mu \alpha$   
 $\partial_\mu \phi^* \partial^\mu \phi = \partial_\mu \rho \partial^\mu \rho + \rho^2 \partial_\mu \alpha \partial^\mu \alpha$

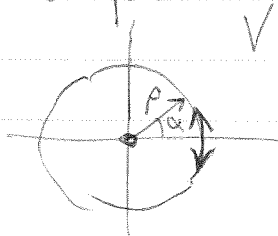
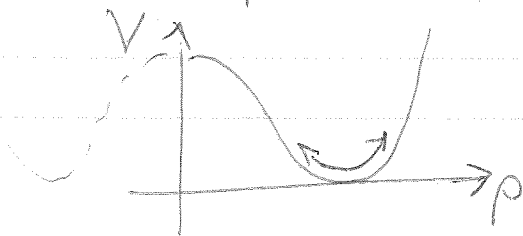
With this, the Lagrangian becomes:

5  $\mathcal{L} = \frac{1}{2} \partial_\mu \rho \partial^\mu \rho + \rho^2 \partial_\mu \alpha \partial^\mu \alpha - \frac{\mu^2}{2} \rho^2 - \frac{\lambda}{4} \rho^4$

around the minima

① Use a perturbation in the  $\rho$  direction and observe the term in  $\mathcal{L}$  corresponding to a mass term  $\Rightarrow$  massive  $\rho$  field (Higgs)

② Use a perturbation in the  $\alpha$  direction and observe now change in the potential  $\Rightarrow$  massless  $\alpha$  field



(c)  $\frac{m^2}{2} A^2 = \frac{m^2}{2} A_\mu A^\mu$  is not  ~~Lorentz invariant~~ gauge invariant ✓  
 local U(1)

$$D_\mu \phi = (\partial_\mu + iA_\mu) \phi = (\partial_\mu + iA_\mu) \rho e^{i\alpha} = e^{i\alpha} \partial_\mu \rho + i\rho e^{i\alpha} \partial_\mu \alpha + iA_\mu \rho e^{i\alpha}$$

$$= e^{i\alpha} \partial_\mu \rho + i\rho e^{i\alpha} (\partial_\mu \alpha + A_\mu)$$

$$D_\mu \phi^* = \dots = e^{-i\alpha} \partial_\mu \rho - i\rho e^{-i\alpha} (\partial_\mu \alpha + A_\mu)$$

↳  $D_\mu \phi D^\mu \phi^* = \partial_\mu \rho \partial^\mu \rho + \rho^2 (\partial_\mu \alpha \partial^\mu \alpha + A_\mu A^\mu + \partial_\mu \alpha A^\mu + A_\mu \partial^\mu \alpha)$

Now, the Lagrangian contains a  $\rho^2 A^2$  term corresponding to the mass of the field  $A_\mu$ .  
 ↳ only if  $g^2$  has value  $> 0$

(d) In that case their masses will be zero, since they are generated by the Higg mechanism

— why, motivate!



6 (a) The delta  $\Delta$  consists of three identical fermions. Its wavefunction should therefore be anti-symmetric. The known quantum number that time, contributed all symmetrically to the wavefunction. In order not to violate the fermi-dirac statistics, an additional quantum number, named colour charge, was introduced that would be anti-symmetric for  $\Delta$ .

$$\psi(\Delta) = \underbrace{\eta(\Delta)}_{\text{space}} \underbrace{\theta(\Delta)}_{\text{space}} \underbrace{\phi(\Delta)}_{\text{space}} \underbrace{\xi(\Delta)}_{\text{colour}}$$



$m(\pi) = 140 \text{ MeV}$

$m(p) = 940 \text{ MeV}$

$m(\Delta) = 1232 \text{ MeV}$

L:  $p(p) = 0 \text{ eV}$

Use the concept of invariant mass:

$$m_{\Delta}^2 = S = (E_{\pi} + E_p)^2 - (p_{\pi} + p_p)^2 = E_{\pi}^2 + E_p^2 + 2E_{\pi}E_p - p_{\pi}^2$$

$$= m_{\pi}^2 + m_p^2 + 2E_{\pi}m_p$$

7

$$p_{\pi} = \sqrt{E_{\pi}^2 - m_{\pi}^2} = \sqrt{\left(\frac{m_{\Delta}^2 - m_{\pi}^2 - m_p^2}{2m_p}\right)^2 - m_{\pi}^2}$$

$$= \sqrt{\left(\frac{1232^2 - 140^2 - 940^2}{2 \cdot 940}\right)^2 - 140^2} \text{ MeV} = 295 \text{ MeV in NU.}$$

	$M = -1$	$M = 0$	$M = 1$	$M = -\frac{3}{2}$	$M = -\frac{1}{2}$	$M = \frac{1}{2}$	$M = \frac{3}{2}$
(c) $\pi$	$\pi^-$	$\pi^0$	$\pi^+$	$\pi^- n$	$\frac{\pi^- p + \pi^0 n}{\sqrt{3}}$	$\frac{\sqrt{2}\pi^0 p + \pi^+ n}{\sqrt{3}}$	$\pi^+ p$
$J = (\frac{1}{2})$		$\otimes$		$J = (\frac{3}{2})$	$\oplus$		
	$M = -\frac{1}{2}$	$M = \frac{1}{2}$		$M = -\frac{1}{2}$	$M = \frac{1}{2}$		
$N$	$n$	$p$		$J = (\frac{1}{2})$	$-\frac{\sqrt{2}\pi^- p + \pi^0 n}{\sqrt{3}}$	$\frac{-\pi^0 p + \sqrt{2}\pi^+ n}{\sqrt{3}}$	
$J = (\frac{1}{2})$							

Use table 1 x 1/2 to find the coefficients

Calculate transition probability:

$$\sigma(\pi^+ p \rightarrow \pi^+ p) \propto |\langle \pi^+ p | \pi^+ p \rangle|^2 = 1$$

$$\begin{aligned} \sigma(\pi^- p \rightarrow \pi^0 n) &\propto |\langle \pi^0 n | \pi^- p \rangle|^2 \\ &\stackrel{*}{=} \left| \langle \pi^0 n | \frac{1}{\sqrt{3}} (\pi^- p + \sqrt{2} \pi^0 n) \right\rangle \left\langle \frac{1}{\sqrt{3}} (\pi^- p + \sqrt{2} \pi^0 n) | \pi^- p \right\rangle \\ &\quad + \left| \langle \pi^0 n | \frac{1}{\sqrt{3}} (-\sqrt{2} \pi^- p + \pi^0 n) \right\rangle \left\langle \frac{1}{\sqrt{3}} (-\sqrt{2} \pi^- p + \pi^0 n) | \pi^- p \right\rangle \\ &= \left| \frac{\sqrt{2}}{3} \right|^2 + \left| \frac{-1}{3} \right|^2 = \frac{2}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3} \end{aligned}$$

\* inserting complete set of commuting operators

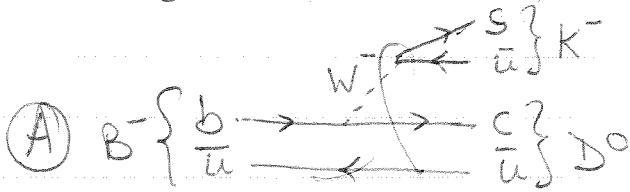
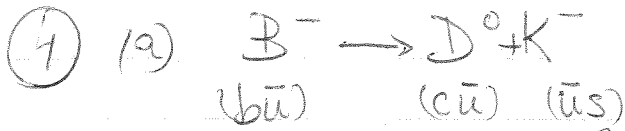
$$\frac{\sigma(\pi^+ p \rightarrow \pi^+ p)}{\sigma(\pi^- p \rightarrow \pi^0 n)} = \frac{|\langle \pi^+ p | \pi^+ p \rangle|^2}{|\langle \pi^0 n | \pi^- p \rangle|^2} = \frac{1}{\frac{1}{3}} = \frac{3}{1} = 3$$

7

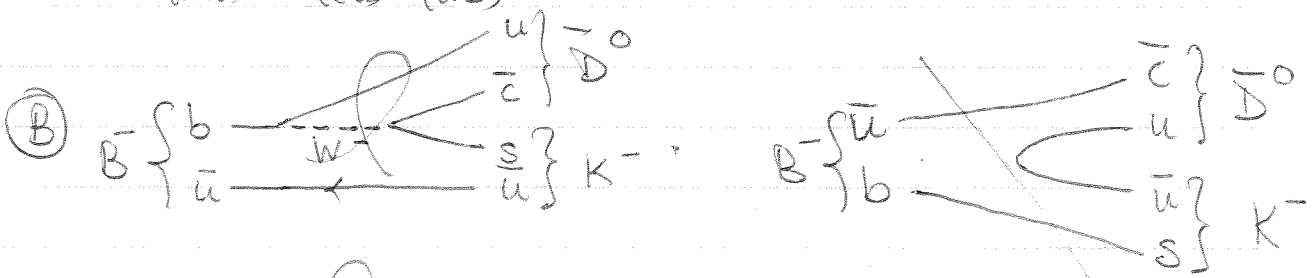
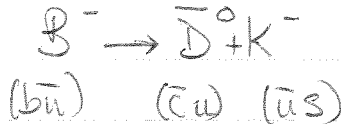
Σ 20  
(3)

13

Q			
$\frac{2}{3}$	u	c	t
$-\frac{1}{3}$	d	s	b



5



(b) (A)  $|V_{cb}| |V_{us}| = 0.0412 \cdot 0.22534 = 0.00928$   
 $b \rightarrow c \quad u \rightarrow s \quad (\rightarrow \bar{u}s)$

(B)  $|V_{ub}| |V_{cs}| = 0.00351 \cdot 0.97344 = 0.00342$   
 $b \rightarrow u \quad c \rightarrow s \quad (\rightarrow \bar{c}s)$

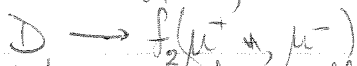
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Probability proportional to CKM matrix coefficients squared times the mass difference to the power 5

$\Rightarrow D^0 (c\bar{u})$  more likely than  $\bar{D}^0 (\bar{c}u)$

(c) Yes, same kind of Feynman diagrams. CPT  
 $\Rightarrow B^{\pm}$  would go likelier to a  $D^0$  than to a  $\bar{D}^0$ .

4

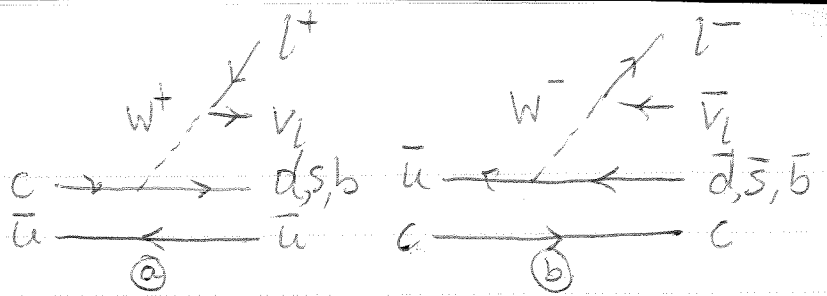


It is much smaller, because the branching fraction is proportional to the mass fraction to the power-five. Since muons are reasonable heavier than electrons, the decay into muons is much smaller.

universality

(e)  $D^0: c\bar{u}$

$\bar{D}^0: \bar{c}u$



probability proportional to square of CKM matrix elements and mass difference to the power five. excellent...  
 relative probability between (a) and (b), gives that (a) is likelier to happen than (b). Therefore if the charge of the lepton is positive, then then it is most likely that it came from  $D^0$ . If on the otherhand the charge of the lepton is negative, it is most likely it came from  $\bar{D}^0$ .

(f)  $CP = +1: |D_1^0\rangle = \frac{1}{\sqrt{2}}(|D^0\rangle + |\bar{D}^0\rangle)$  (1)

$CP = -1: |D_2^0\rangle = \frac{1}{\sqrt{2}}(|D^0\rangle - |\bar{D}^0\rangle)$  (2)

$\Rightarrow |D^0\rangle \stackrel{(1)+(2)}{\frac{2}{\sqrt{2}}} = \frac{1}{\sqrt{2}}(|D_1^0\rangle + |D_2^0\rangle)$

12/2

⑤ (a) Similarities:

- mixing matrices
- 3x3 matrices
- change bases.

5

Differences:

- for neutrinos and for quarks
- number of free parameters (depends on the model)

Mixing angles  $\theta_{12}, \theta_{23}, \theta_{13}, \delta$

(b) According to the theory, the expression for the oscillation behaviour contains a sine term that contains a mass difference term,  $\Delta m$ , between the neutrino masses. (Some-thing of the form  $\sin(\frac{\Delta m^2}{2}t)$ ) If oscillation is observed, this implies then that  $\Delta m$  is non-zero, therefore neutrinos have a mass.

The condition that the theory is correct must be met.

↳ true ....  $\frac{1}{2}$



One can use conservation of four momentum (momentum and energy).

One knows the masses of the  $\pi^\pm$  and  $\mu^\pm$  rather precisely, one can measure the momenta of the  $\mu^\pm$ ,  $\nu_\mu/\bar{\nu}_\mu$ . From this, one can obtain an estimate on the neutrino mass.

Depends on the known accuracy of the masses of the pion and the kaon. Since the kaon is heavier, it can attain more momentum, therefore making the estimate more precise. Prefer kaon use.